A warping model for reinforced concrete beams under multiaxial loading

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Abstract

The present research focuses on the numerical modelling of structures using the multifiber beam element method. A formulation to account for shear warping is developed. The local warping degrees of freedom are computed along with the global beam degrees of freedom by satisfying the structure equilibrium. In the case of pure torsion, the evolution of the warping shape only depends on the concrete damage state. The comparison of numerical and experimental torque-twist curves ascertains the importance of including torsional warping in the formulation. Ongoing work tests the numerical behaviour of structural elements subject to transverse shear warping.

Keywords : torsion, warping, concrete, damage, nonlinear analysis, finite elements, multifiber beams

1 Introduction

Assessing the seismic vulnerability of existing reinforced concrete structures by a nonlinear time analysis requires a large number of structural degrees of freedom and time steps. Computational efficiency is thus crucial in the choice of the numerical method. Based on beam elements, multifiber elements have been developed to combine the advantages of the high computational speed with an increased accuracy for the nonlinear materials. As in the classical beam element method, displacements and forces are computed at the beam element nodes. Generalized strains are computed at the beam element Gauss points. Then, instead of using a generalized constitutive law, a two-dimensional domain embodying the beam's cross section is considered. Actual strains in the cross section are computed using Euler-Bernoulli or Timoshenko's assumption for the cross section motion. Stresses are obtained from strains through the chosen constitutive laws, which allows for various materials or material states in the cross section. Finally, the generalized forces are obtained by integration of the stresses over the cross section.

The multifiber beam element method has been successfully applied to slender structural elements subject to bending and/or axial loading (Grange et al., 2008). However, the method fails to compute the behaviour of structures subject to large shear stresses. The beam theory approximation of a plane cross section is not accurate enough to convey the effect of shear. To address this issue, several authors have enhanced the multifiber beam elements by adding warping deformations. A fixed warping pattern can be used, such as proposed by Dubé (1997) or Mazars et al. (2006). Other contributions update the warping profile throughout the computation by solving a coupled equilibrium problem. Classical beam degrees of freedom are computed considering the global beam equilibrium, while additional warping degrees of freedom are computed by satisfying the equilibrium of the cross section. Based on this approach, Bairan and Mari (2007) formulated a cross section stiffness matrix and resisting force vector accounting for warping and in-plane distortion of the cross section. Le Corvec (2012) developed a complete force-based beam element with additional warping deformations for an elasto-plastic material.

The present research proposes a formulation for displacement-based multifiber beam elements, coupling the evolution of warping in the cross section with the state of damage of the material. The theoretical formulation to take into account warping due to transverse shear and torsion is presented first. Then, the method is simplified in the case of pure torsion. As an example of numerical results, this paper presents the influence of torsional warping on the global behaviour.

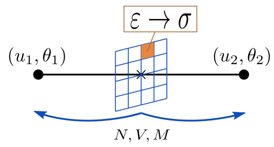

Figure 1: Multifiber beam element principle

2 Formulation of the model with warping

2.1 Beam under multiaxial loading

A warping displacement field $\mathbf{u}^w$ is added to the displacements of the plane section $\mathbf{u}^p$ obtained from Timoshenko's beam theory (equation 1). Warping is only considered in the axial direction, for the sake of computational efficiency. The linearised strain tensor inside the beam is computed from the total displacement field as provided in equation 2. Equation 3 expresses the nonlinear constitutive law.

$$\mathbf{u} = \mathbf{u}^p + \mathbf{u}^w = \begin{bmatrix} u_m - y\,\theta_z + z\,\theta_y \\ v_m - z\,\theta_x \\ w_m + y\,\theta_x \end{bmatrix} + \begin{bmatrix} u_x^w(x,y,z) \\ 0 \\ 0 \end{bmatrix} \tag{1}$$

$$\boldsymbol{\varepsilon} = \frac{1}{2}\left(\mathbf{grad}(\mathbf{u}) + \mathbf{grad}(\mathbf{u})^T\right) = \boldsymbol{\varepsilon}^p + \boldsymbol{\varepsilon}^w \tag{2}$$

$$\boldsymbol{\sigma} = \hat{\boldsymbol{\sigma}}\left(\boldsymbol{\varepsilon}^p, \boldsymbol{\varepsilon}^w\right) \tag{3}$$

Beam equilibrium is written in its weak form and projected on the two subspaces of plane section displacements and warping displacements, assumed to be orthogonal. Equations 4 and 5 are obtained after integration by part of the two equilibrium equations. F denotes the external forces and P^w the forces coming from constrained warping at the beam ends.

$$\int_{\Omega} \delta \boldsymbol{\varepsilon}^{pT} \hat{\boldsymbol{\sigma}} \left(\boldsymbol{\varepsilon}^{p}, \boldsymbol{\varepsilon}^{w} \right) \; d\Omega = F \tag{4}$$

$$\int_{\Omega} \delta \boldsymbol{\varepsilon}^{wT} \hat{\boldsymbol{\sigma}} \left(\boldsymbol{\varepsilon}^{p}, \boldsymbol{\varepsilon}^{w} \right) \; d\Omega = P^{w} \tag{5}$$

After discretization of the beam and the section domains, the formulation leads to two coupled systems of nonlinear equations to solve for the classical beam nodal displacements and for the additional warping displacements. By linearising equations 4 and 5, and assuming unconstrained warping ($P^w = 0$), we get the following linearised system of equations:

$$\begin{bmatrix} A_{pp} & A_{pw} \\ A_{wp} & A_{ww} \end{bmatrix} \begin{bmatrix} \bar{\mathbf{u}}^{timo} \\ \bar{\mathbf{u}}^{w} \end{bmatrix} = \begin{bmatrix} F \\ 0 \end{bmatrix} \tag{6}$$

The warping degrees of freedom can either be treated as additional global degrees of freedom of the beam element or condensed out to be treated as local degrees of freedom in the cross section. To deal with computations at a structural scale, it is preferable to consider the warping displacements as local degrees of freedom, and to independently treat each cross section.

2.2 Simplification in the case of pure torsion

This paragraph shows how the formulation detailed in paragraph 2.1 naturally simplifies in the case of pure torsion. The plane cross section displacements $\mathbf{u}^p$ are provided in equation 7. The warping displacement shape is given in equation 8.

$$\mathbf{u}^p = \begin{bmatrix} 0 \\ -z\,\theta_x \\ y\,\theta_x \end{bmatrix} \tag{7}$$

$$\mathbf{u}^w = \begin{bmatrix} \frac{\theta_x}{L}\varphi(y,z) \\ 0 \\ 0 \end{bmatrix} \tag{8}$$

Taking into account the simplification of the displacements in the pure torsion case and assuming that the stress-strain relationship is linear over a load increment, equation 5 simplifies into equation 9. G denotes the material shear modulus.

$$\int_{\mathcal{S}} \left[\frac{\partial \delta\varphi}{\partial y} G \left(\frac{\partial \varphi}{\partial y} - z \right) + \frac{\partial \delta\varphi}{\partial z} G \left(\frac{\partial \varphi}{\partial z} + y \right) \right] d\mathcal{S} \tag{9}$$

Equation 9 shows that the warping function φ only depends on the geometrical and material properties of the cross section. The local equation is uncoupled from the global equilibrium equation. Consequently, the warping function is computed at each converged time step according to the updated material shear modulus. Warping-induced shear strains are then added to the plane section strains to compute the nodal displacements by satisfying the beam global equilibrium.

3 Results

To examine the influence of warping on the behaviour of concrete structures modelled by multifiber beam elements, a series of tests was conducted in pure torsion first. The experimental tests carried out by Chalioris and Karayannis (2009) on plain concrete beams of length $L = 1.30\,\text{m}$ were chosen as a reference. The specimens were discretized into 4 multifiber beam elements, with and without the additional torsional warping deformations. They were loaded in axial rotation, by 50 steps of $0.03\,\text{rad}$. The rectangular, L-shaped and T-shaped cross sections were meshed using linear triangles, as displayed in figure 2. In the cross section elements, concrete was modelled using the Mu damage model, (Mazars et al., 2013). With the nonlinear material parameters previously calibrated using a genetic algorithm, the Young modulus was computed for each case study from the experimental compressive strength using the European design code. Around this mean value, the Young modulus was modelled as a normally distributed random field in each cross section, to represent the material spatial variability.

Figure 2 displays the torque-twist curves obtained by both models with and without warping. For each selected cross section shape, the numerical computations were performed using 5 different realizations for the random Young modulus distribution. In figure 2, the continuous lines represent the result means, while the dashed lines account for the result standard deviation.

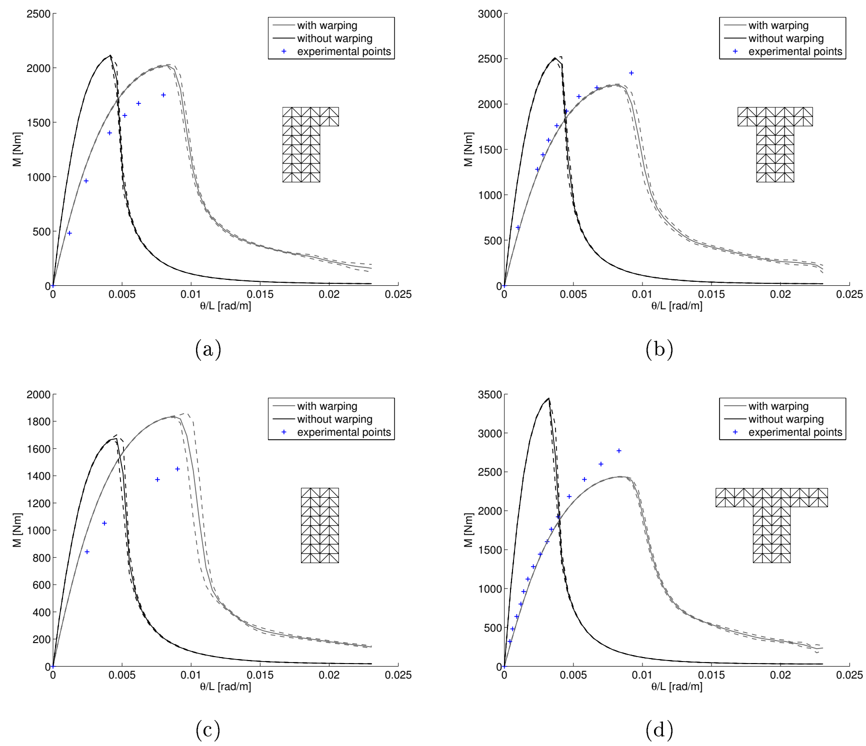

Figure 2: Torque-twist curves obtained by prediction with the numerical model, comparison with experimental behaviour. 2(a) Ls-shaped cross section; 2(b) Ts-shaped cross section; 2(c) rectangular cross section; 2(d) T-shaped cross section.

The initial stiffness is well predicted by the model with warping, while it is overestimated by the model without warping. By taking warping into account, there is an improvement of 90% on the initial stiffness prediction. Considering both ultimate torque and twist values, the prediction of the torque-twist peak is improved by 70% if the cross section is allowed to warp. This result confirms the importance of including warping in the multifiber element formulation for beams in torsion.

4 Conclusion

This paper proposes a method to improve the results of multifiber beam elements to model concrete structures subject to large shear stresses. Warping displacements of the cross section are added to the displacements of the plane cross section. The equilibrium of the enriched beam leads to a coupled system of equations to solve for the structure. The model can be simplified in the case of pure torsion. The formulation is used to carry out multifiber finite elements computations for plain concrete rectangular, T-shaped and L-shaped beams in torsion. By taking warping into account, the prediction of the initial stiffness is improved by 90% and the prediction of the peak value by 70%, at the cost of 77% more computational time. To further analyse the effect of warping, the computation of reinforced concrete structural elements subject to multiaxial loading modelled by the complete warping formulation is subject of current work.

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